Learn the concept of Factors and Multiples for WBCS Exam

In the realm of competitive exams, the knowledge of **Factors and Multiples** forms the bedrock of mathematical reasoning. Whether you're aspiring to conquer the **WBCS Exam** or any other competitive test, understanding these fundamental concepts is vital. Factors are the building blocks of numbers, and multiples are their extensions, intricately linked in mathematical relationships. This guide will unravel the intricacies of Factors and Multiples, providing you with the tools to tackle numerical questions with confidence.

From grasping the essence of prime factors to discerning the properties of multiples, we will explore every facet of this topic. By the end of this journey, you'll not only have a solid grasp of these concepts but also the ability to apply them effectively in the WBCS Exam, propelling you towards success in your competitive endeavors. So, let's embark on this mathematical expedition to sharpen your skills and enhance your exam readiness.

This is the third part of the '**Number System**' blog series. So, before understanding concepts of factors and multiples, first understand the 'Classification of Numbers' and 'Divisibility Rule of Numbers' by clicking the embedded links. Now, let's start understanding the concepts of factors and multiples.

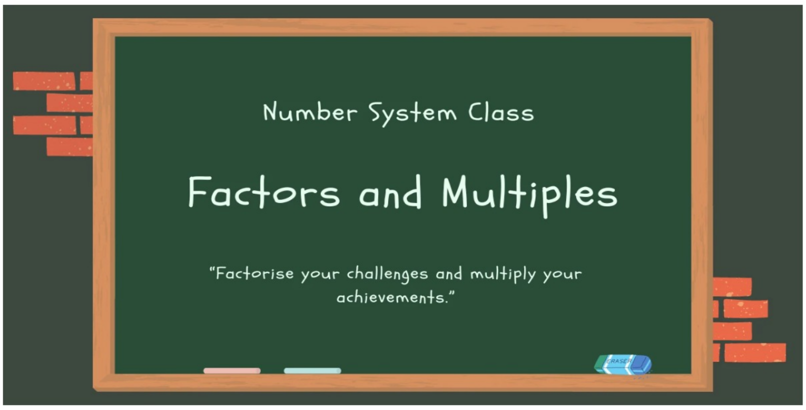
Factors and Multiples

Firstly, let's just differentiate between the meaning of factors and multiples.

FACTORS: Factors of a number are the numbers by which when the number is divided, the remainder will be 0.

MULTIPLES: Multiples of a number are the numbers which are if divided by the number, the remainder will be 0.

E.g. 8 has the factors 1, 2, 4 and 8 and has the multiples 8, 16, 24, etc.



Now, let's talk about factors.

Properties of Factors

We have learned in the first blog that any number greater than 1 can be written as the product of prime factors. Let's generalize this term then we will explore this topic.

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

Let N is a composite number. So, by above statement, N can be written as,

$N = a^p \times b^q \times c^r \times ...$

(Where a, b, c are distinct prime factors of N and p, q, r are integers)

So, the general formula to calculate the number of factors of N is,

Total number of factors of N = (p + 1) (q + 1) (r + 1) ...

Let's take an example.

E.g. Find the total number of factors of 756.

Sol: 756 can be written as, $756 = 2^2 \times 3^3 \times 7$

So, the total number of factors of 756 = (2 + 1) (3 + 1) (1 + 1) = 3 × 4 × 2 = 24. (**Ans.**)

E.g. How many factors of 756 are multiple of 12?

Sol: $756 = 2^2 \times 3^3 \times 7$ and $12 = 2^2 \times 3$

$\Rightarrow 756 = 2^2 \times 3^1 [3^2 \times 7^1]$

Since we have to find out the multiples of 12 in it. So, this $[3^2 \times 7^1]$ is important for us.

No. of factors = (2 + 1) (1 + 1) = 3 + 2 = 6. (**Ans.**)

Let's look at the properties of Factors. The properties of factors are as follows:

- Every number has at least two factors: 1 and itself.
- A prime number has only two factors: 1 and itself.
- Every factor of a number is less than or equal to the number itself.
- The product of two factors of a number is always equal to the number itself.
- The sum of two factors of a number is not always equal to the number itself.
- The number of factors of a number is finite.
- The factors of a product are the same as the product of the factors of the individual numbers.
- The factors of a sum are not the same as the sum of the factors of the individual numbers.

Note: The number of factors of a perfect square is always odd because if we write the factors of any number in ascending order then the product of corresponding terms will give the number itself.

E.g. Factors of 100 = 1, 2, 4, 5, 10, 20, 25, 50, 100.

Here, 1 × 100 = 100, 2 × 50 = 100, 4 × 25 = 100, 5 × 20 = 100.

But since only 10 is left so it should be multiply by itself (that’s why 100 is perfect square). So that’s why, the number of factors of a perfect square is always odd.

We've discussed Co-prime numbers in our previous blogs. Let's discuss Co-prime numbers in detail.

Co-Prime Numbers

As discussed earlier, Co-Prime numbers are the two numbers which have no common factor other than 1.

E.g. 24 and 25 are co-prime numbers because 24 has factors 1, 2, 3, 4, 6, 8, 12, 24 and 25 has factors 1, 5, 25. No factor is common other than 1.

Now let us take a look at some interesting points:

1. The product of two consecutive integers is always even. Because one of them is even and one of them is odd, so the product will always be even.
2. The product of three consecutive integers is always divisible by 6. Because one of them is always even and one of them is always divisible by 3.
3. Like that, the product of four consecutive integers is always divisible by 24.
4. The product of n consecutive integers will always be divisible by n!
5. 1 digit numbers are 9.
   2 digit numbers are 90 (from 10 to 99). So, they have 90 × 2 = 180 digits.
   3 digit numbers are 900 (from 100 to 999). So, they have 900 × 3 = 2700 digits.
   4 digit numbers are 9000 (from 1000 to 9999). So, they have 9000 × 4 = 36000 digits. And so on.

E.g. Divya typed the first n natural numbers on a keyboard without any spaces. If she had to press keys 1692 times, find n.

Sol: 1 digit numbers have 9 digits. 2 digit numbers are 90 and have 180 digits. So, 1692 – (180 + 9) = 1503.

Since 3 digit numbers are 900 and have 2700 digits but we left only 1503 digits, it means the number is a 3 digit number.

Now, 1503/3 = 501, means it is the 501th 3 digit number.

So, the number is, 501 + 90 + 9 = 600. (**Ans.**)

Now, let's look at the Multiples. It's not something very hard to discuss but let's look at the properties of Multiples.

Properties of Multiples

As discussed earlier, a multiple of a number is a product of that number and any other integer.

For example, the multiples of 5 are 5, 10, 15, 20, 25, and so on.

Let's look at some properties of Multiples.

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

- Every integer is a multiple of itself and 1.
- The multiples of a number are always greater than or equal to the number itself.
- There is no end to the multiples of a number.
- The multiples of an odd number are alternatively odd and even.
- The sum of two or more multiples of a number is also a multiple of that number.
- The product of two or more multiples of a number is also a multiple of that number.

We will discuss the Factors and Multiples in detail in our next blog i.e. **LCM and HCF**. But for now let us look at few examples and if you are now confident about this section, just try these questions by yourself before looking at my solution. It would be fun. Some of these questions were asked in the WBCS Exam. So, please take this section very seriously.

Examples on Factors and Multiples for Better Understanding

E.g. The sum of first five multiples of 3 is **[WBCS Exam 2021]**

Sol: The first five multiples of 3 are 3, 6, 9, 12, 15.

Their sum = 3 + 6 + 9 + 12 + 15 = 45. (**Ans.**)

E.g. The number of multiples of 4 between 10 and 250 is **[WBCS Exam 2021]**

Sol: The number of multiples of 4 below 250 = 250/4 = 62.5 ≈ 62 multiples

The number of multiples of 4 below 10 = 10/4 = 2.5 ≈ 2 multiples

Required answer = 62 - 2 = 60. (**Ans.**)

E.g. The sum of three consecutive multiples of 3 is 72. What is the largest number? **[WBCS Exam 2018]**

Sol: Let, the three consecutive multiples of 3 are 3(a - 1), 3a, 3(a + 1).

So, 3(a - 1) + 3a + 3(a + 1) = 72

⇒ 3a - 3 + 3a + 3a + 3 = 72

⇒ 9a = 72

⇒ a = 8

So, the three numbers are (3 × 7), (3 × 8) and (3 × 9) i.e. 21, 24, 27.

Hence, the largest number is 27. (**Ans.**)

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

Now, let's take 2 high level very interesting questions of this topic which will help you to understand this topic and its applications in deep.

E.g. How many natural numbers divide 35^{999} but do not divide 35^{998}?

Sol: Just read the question 2 or 3 times and find what this question wants us to calculate. It wants us to find the factors of 35^{999} which are not the factors of 35^{998}. In simple words,

(No. of factors of 35^{999}) – (No. of factors of 35^{998}),

Because these are the only factors which are greater than 35^{998} and, so that, they do not divide 35^{998}.

Now,

$\Rightarrow 35^{999} = (5 \times 7)^{999} = 5^{999} \times 7^{999}$

$\Rightarrow$ No. of factors = (999 + 1) (999 + 1) = 1000 × 1000 = 10002

Also,

$\Rightarrow 35^{998} = (5 \times 7)^{998} = 5^{998} \times 7^{998}$

$\Rightarrow$ No. of factors = (998 + 1) (998 + 1) = 999 × 999 = 9992

$\therefore$ required answer = 10002 – 9992 = (1000 + 999) (1000 – 999) = 1999 × 1 = 1999. (**Ans.**)

E.g. In an NGO, there are 70 workers. All the workers visited an old age home having 70 senior citizens. The first worker donated Rs. 1000 to each senior citizen. The second worker donated Rs. 1000 to every second senior citizen starting from the second senior citizen. The third worker donated Rs. 1000 to every third senior citizen starting from the third senior citizen and so on. How many senior citizens received donations from an odd number of workers?

Sol: Read the question twice and thrice and try to understand what this question wants to say. The first worker gives rupees to all 70 citizens. But the 2nd worker gives only to 2nd, 4th, 6th, 8th, etc. citizens. Same thing is done by 3rd, he gives rupees to only 3rd, 6th, 9th, 12th, etc. citizens. Same for 4th, 5th, 6th workers till 70th.

It means, a worker whose number is x is giving money to all the citizens whose number is a multiple of x.

And hence, the citizen, whose number is y, is receiving the donations from all the workers whose number is a factor of y. i.e. is receiving as many donations as the number of factors of y.

So, we just have to find out, from 1 to 70, how many numbers have odd numbers of factors.

And from the previous discussion, we know that only perfect squares have an odd number of factors.

And from 1 to 70, there are 8 perfect squares, which are 1, 4, 9, 16, 25, 36, 49 and 64.

So, the answer is, only 8 senior citizens received donations from an odd number of workers. (**Ans.**)

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

In conclusion, mastering the concept of Factors and Multiples is a pivotal step towards WBCS Exam success. These mathematical foundations are essential for problem-solving and logical reasoning. Armed with this knowledge, you're better equipped to navigate the numerical challenges of the exam and elevate your chances of achieving your goals.

So, this is all for this blog. We will discuss the LCM and HCF in our next blog of this 'Number System' blog series. Till then, keep practicing!

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com